

# True radiation gauge for gravity

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## Abstract

Corresponding to the similarity between the Lorentz gauge  $\partial_\mu A^\mu = 0$  in electrodynamics and  $g^{\mu\nu}\Gamma_{\mu\nu}^\rho = 0$  in gravity, we show that the counterpart of the radiation gauge  $\partial_i A^i = 0$  is  $g^{ij}\Gamma_{ij}^\rho = 0$ , instead of other forms as discussed before. Particularly: 1) at least for a weak field,  $g^{ij}\Gamma_{ij}^\rho = 0$  fixes the gauge completely and picks out exactly the two physical components of the gravitational field; 2) like  $A^0$ , the non-dynamical components  $h_{0\mu}$  are solved instantaneously; 3) gravitational radiation is generated by the “transverse” part of the energy-momentum tensor, similar to the transverse current  $\vec{J}_\perp$ . This “true” radiation gauge  $g^{ij}\Gamma_{ij}^\rho = 0$  is especially pertinent for studying gravitational energy, such as the energy flow in gravitational radiation. It agrees with the transverse-traceless (TT) gauge for a pure wave, and reveals remarkably how the TT gauge can be adapted in the presence of source.

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Gauge invariance is a powerful guidance in building field theories, but can be a nuisance in actual calculations. Its advantage and disadvantage are just the two sides of the same coin: The redundant field components help to write down an elegant Lagrangian, but have to be got rid of when identifying the real physical degrees of freedom. In electrodynamics, one uses a four-component vector field  $A^\mu$  to describe a massless photon with two physical polarizations. The radiation (or Coulomb, transverse) gauge  $\vec{\partial} \cdot \vec{A} = 0$  can perfectly remove the gauge freedom and specify the transverse field  $\vec{A}_\perp$  to be the two physical components which propagate in electromagnetic radiation. In general relativity, one uses a ten-component symmetric tensor  $g_{\mu\nu}$  to describe the gravitational field, but the number of physical degrees of freedom is still two, thus, naturally, the gauge fixing problem is much harder. A gravitational “radiation gauge” as satisfactory as  $\vec{\partial} \cdot \vec{A} = 0$  in electrodynamics should meet the following criteria:

1. It contains four and only four realizable constraints.
2. It fixes the gauge completely, and picks out exactly the two physical components of the gravitational field.

So far as we know, such a gauge has not yet been reported. In this paper, we show that it does exist, at least for linearized gravity. It differs remarkably from other gauges discussed before, and leads to interesting implications concerning gravitational radiation and gravitational energy.

When handling gravitational radiation, the most frequently used gauge is the harmonic or De Donder gauge  $g^{\mu\nu}\Gamma_{\mu\nu}^\rho = 0$ . It plays a similar role as does the Lorentz gauge  $\partial_\mu A^\mu = 0$  in electrodynamics. In the weak-field approximation,  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$  with  $\eta_{\mu\nu}$  the Minkowski metric and  $|h_{\mu\nu}| \ll 1$ , the harmonic gauge becomes

$$\partial^\mu h^\rho{}_\mu - \frac{1}{2}\partial^\rho h^\mu{}_\mu = 0. \quad (1)$$

(In the linear approximation, indices are raised and lowered with the Minkowski metric, Greek indices run from 0 to 3, Latin indices run from 1 to 3, and repeated indices are summed over.) By Eq. (1), the linearized Einstein equation,

$$\square h_{\mu\nu} - \partial_\mu \partial_\rho h^\rho{}_\nu - \partial_\nu \partial_\rho h^\rho{}_\mu + \partial_\mu \partial_\nu h^\rho{}_\rho = -S_{\mu\nu}, \quad (2)$$

reduces to the familiar form with a retarded solution:

$$\square h_{\mu\nu} = -S_{\mu\nu}. \quad (3)$$

Here  $\square \equiv \vec{\partial}^2 - \partial_t^2$ ,  $S_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^\rho{}_\rho$ , and we put  $16\pi G = 1$ .

The harmonic gauge, however, like the Lorentz gauge  $\partial_\mu A^\mu = 0$ , does not fix the gauge completely, and is not sufficient to specify the two physical polarizations of the gravitational wave. For pure gravitational waves without matter source, it was found that the gauge freedom can be completely removed by the transverse-traceless (TT) gauge [1]:

$$h_{0\mu} = 0, \quad (4a)$$

$$\partial_i h^i{}_j = 0, \quad (4b)$$

$$h^i{}_i = 0. \quad (4c)$$

Nonetheless, as Ref. [1] clearly illustrates, *only pure waves (and not more general solutions of the linearized field equations with source) can be reduced to TT gauge*. This can be easily seen by counting the number of constraints: General relativity admits a four-fold freedom of coordinate transformation, but the TT gauge has altogether eight independent constraints, thus can be satisfied only for special cases such as a pure wave, but not for general cases. [2] In fact, somehow surprisingly, even for the linearized gravity we cannot find in the literature a fully satisfactory gauge condition. Sometimes, the gauge  $\partial^i h^\rho{}_i = 0$  is called the ‘‘Coulomb gauge’’. But just like that  $\partial^\mu h^\rho{}_\mu = 0$  does not play a similar role in gravity as  $\partial_\mu A^\mu = 0$  does in electrodynamics,  $\partial^i h^\rho{}_i = 0$  in gravity is not as convenient as  $\partial_i A^i = 0$  in electrodynamics.

Resembling the relation between the Lorentz gauge  $\partial_\mu A^\mu = 0$  and the radiation gauge  $\partial_i A^i = 0$ , we find that by a simple generalization from the harmonic gauge,  $g^{\mu\nu}\Gamma^\rho{}_{\mu\nu} = 0$ , we can get the ‘‘true’’ radiation gauge for gravity:  $g^{ij}\Gamma^\rho{}_{ij} = 0$ , or in linearized form:

$$\partial^i h^\rho{}_i - \frac{1}{2}\partial^\rho h^i{}_i = 0. \quad (5)$$

This contains exactly four constraints, and we can check that they can always be imposed: If  $h_{\mu\nu}$  does not satisfy Eq. (5), we can find a gauge-transformed  $h'_{\mu\nu}$  that does. Under an infinitesimal coordinate transformation:  $x^\mu \rightarrow x'^\mu = x^\mu - \epsilon^\mu(x)$ , with  $\epsilon^\mu$  four arbitrary infinitesimal functions,  $h_{\mu\nu}$  undergoes a gauge transformation,

$$h_{\mu\nu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu. \quad (6)$$

The goal can be achieved by choosing  $\epsilon^\mu$  with

$$\vec{\partial}^2 \epsilon^\rho = -(\partial^i h^\rho_i - \frac{1}{2} \partial^\rho h^i_i). \quad (7)$$

By defining  $\mathfrak{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h^\rho_\rho$ , the harmonic gauge can be put in a concise Lorentz-like form  $\partial^\mu \mathfrak{h}_\mu^\rho = 0$ . Similarly, if we define  $\mathfrak{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h^k_k$ , Eq. (5) can be put into a concise transverse condition

$$\partial^i \mathfrak{h}^\rho_i = 0. \quad (8)$$

It must be noted, however, that  $\partial^i \mathfrak{h}^\rho_i = 0$  does not lead to the desired gauge (5). This might partially explain why the gauge (5) has eluded people's attention.

It can be seen in two ways that Eq. (5) removes all non-physical degrees of freedom of the gravitational field:

- (i) Eq. (5) permits no more gauge freedom.
- (ii) Eq. (5) leads to the equation of motion by which only two physical components propagate.

Proof of (i): Under a gauge transformation in (6), preservation of the gauge condition (5) requires

$$\partial^i (\partial^\rho \epsilon_i + \partial_i \epsilon^\rho) - \frac{1}{2} \partial^\rho (\partial_i \epsilon^i + \partial_i \epsilon^i) = \vec{\partial}^2 \epsilon^\rho = 0. \quad (9)$$

By the boundary condition that  $\epsilon^\rho \rightarrow 0$  at infinity [3], the solution is  $\epsilon^\rho \equiv 0$ . With such boundary condition, the solution to Eq. (7) is also fixed:

$$\epsilon^\rho = -\frac{1}{\vec{\partial}^2} (\partial^i h^\rho_i - \frac{1}{2} \partial^\rho h^i_i). \quad (10)$$

Namely, the gauge transformation that brings an arbitrary (weak) field tensor to the desired gauge in Eq. (5) is unique. Incidentally, this suggests a gauge-invariant construction:

$$\hat{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{\vec{\partial}^2} (\partial_\mu \partial^i h_{i\nu} + \partial_\nu \partial^i h_{\mu i} - \partial_\mu \partial_\nu h^i_i). \quad (11)$$

Proof of (ii): Apply the gauge (5), a careful examination reveals that the gravitational equations of motion and gravitational radiation resemble exactly the form of electrodynamics in the radiation gauge:

$$\vec{\partial}^2 h_{0\mu} = -S_{0\mu}, \quad (12a)$$

$$\square h_{ij} + \partial_t (\partial_j h_{0i} + \partial_i h_{0j}) - \partial_i \partial_j h_{00} = -S_{ij}. \quad (12b)$$

To appreciate these equations and cast them into more elucidating forms, let us recall the corresponding ones in electrodynamics. In the radiation gauge  $\partial_i A^i = 0$ , the Maxwell equations  $\partial_\mu F^{\mu\nu} = -j^\nu$  take the form

$$\vec{\partial}^2 A^0 = -j^0, \quad (13a)$$

$$\square A^i - \partial_t \partial^i A^0 = -j^i. \quad (13b)$$

Solving  $A^0$  by Eq. (13a), and using the conservation condition  $\partial_\mu j^\mu = 0$ , Eq. (13b) can be casted into

$$\square \vec{A}_\perp = -\vec{j}_\perp, \quad (14)$$

where  $\vec{j}_\perp = \vec{j} - \vec{\partial} \frac{1}{\vec{\partial}^2} (\vec{\partial} \cdot \vec{j})$  is the transverse part of the electric current. Eq. (14) has a very clear physical meaning: It is the transverse field  $\vec{A}_\perp$  with two independent components that propagate in electromagnetic radiation, with the transverse current  $\vec{j}_\perp$  as its source.

Remarkably, we find that Eq. (12b) can be casted into a form analogous to Eq. (14):

$$\square \hat{h}_{ij} = -\hat{S}_{ij}. \quad (15)$$

Here we have put a hat on  $h_{\mu\nu}$  to remind that it is in the radiation gauge and satisfies  $\partial^i \hat{h}^\rho_i - \frac{1}{2} \partial^\rho \hat{h}^i_i = 0$ . [It is also consistent to take this  $\hat{h}_{\mu\nu}$  as the gauge-invariant quantity in Eq. (11)]. Like  $\vec{A}_\perp$ ,  $\hat{h}_{ij}$  has also only two (six minus four) independent components. They are the true dynamical components that propagate in gravitational radiation. And similar to  $\vec{j}_\perp$ , the radiation source  $\hat{S}_{ij}$  is given by

$$\hat{S}_{ij} = S_{ij} - \frac{1}{\vec{\partial}^2} (\partial_i \partial_k S^k_j + \partial_j \partial_k S^k_i - \partial_i \partial_j S^k_k). \quad (16)$$

The derivation of Eqs. (15) and (16) is exactly analogous to that in electrodynamics: Solving  $h_{0\mu}$  by Eq. (12a), and using the conservation condition in the weak-field approximation,  $\partial_\mu T^\mu_\nu = 0$ , which implies  $\partial_\mu S^\mu_\nu = \frac{1}{2} \partial_\nu S^\mu_\mu$ . As a critical cross check, it can be verified by Eq. (16) that

$$\partial_i \hat{S}^i_j - \frac{1}{2} \partial_j \hat{S}^i_i = 0, \quad (17)$$

in consistent with  $\partial^i \hat{h}^\rho_i - \frac{1}{2} \partial^\rho \hat{h}^i_i = 0$  in Eq. (15). Similar to  $\mathfrak{h}_{\mu\nu}$ , we can define  $\mathfrak{S}_{\mu\nu} = S_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} S^k_k$  (again, *not*  $\mathfrak{S}_{\mu\nu} = S_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} S^\rho_\rho$ ), and put Eq. (17) into a transverse condition  $\partial_i \hat{\mathfrak{S}}^i_j = 0$ . Then, similar to Eq. (14), Eq. (16) can be (loosely) interpreted as that the gravitational radiation is generated by the “transverse” part of the energy-momentum tensor.

The constructions in Eqs. (11) and (16) are exactly the same. In two other papers of this serial study [4, 5], we show that such construction applies quite generally to any symmetric tensor field, and can be derived by seeking a unique decomposition of a tensor field into physical and pure-gauge components, analogous to the recent physical decomposition of the Abelian and non-Abelian gauge fields by Chen and collaborators [6, 7].

The “true” radiation gauge we find has interesting relation to, but crucial advantage over, the harmonic and TT gauges. This can be most clearly seen for the pure wave solution in the vacuum. In the harmonic gauge, the equations of motion (3) set all ten field components to propagate, while the gauge conditions only reduce the number of independent components to six. Thus, to obtain the two physical polarizations, four additional constraints have to be imposed by hand. The TT gauge, on the other hand, can promptly pick out the two physical polarizations, but at the price of “brutally” imposing eight gauge conditions (which exceed the actual number of gauge freedom and do not apply in general). The exact counterpart of the TT gauge in electrodynamics is

$$A^0 = 0 \tag{18a}$$

$$\vec{\partial} \cdot \vec{A} = 0 \tag{18b}$$

which, again, picks out the two physical polarizations of a free electromagnetic field by “brutally” imposing more constraints than the actual number of gauge freedom, and cannot be imposed in general.

In our true radiation gauge, however, four field components ( $h_{0\mu}$ ) are determined instantaneously by the source and are non-dynamic, just like  $A^0$  in electrodynamics. Especially, in the vacuum with  $S_{\mu\nu} = 0$ , we have automatically  $h_{0\mu} = -\frac{1}{\partial^2} S_{0\mu} \equiv 0$  by the trivial boundary condition that  $h_{0\mu}$  vanish at infinity. Then, of the six field components  $h_{ij}$  that propagate, the four gauge conditions in Eq. (5) precisely pick out the two physical ones.

A closer look can reveal that with  $h_{0\mu} = 0$ , Eq. (5) can reproduce Eqs. (4b) and (4c): Set  $\rho = 0$  in Eq. (5), we have  $\partial^0 h^i_i = 2\partial^i h^0_j = 0$ , thus the spatial trace  $h^i_i$  is static, and actually must be identically zero by the wave equation in the vacuum,  $\square h_{ij} = 0$ , together with a trivial boundary condition. Then by setting  $\rho = j$  in Eq. (5) we get  $\partial^i h^j_i = \frac{1}{2}\partial^j h^i_i = 0$ , which is just Eq. (4b). Namely, for a pure wave in our true radiation gauge, we get the same eight constraints as the TT gauge by only four gauge conditions plus four equations of motion. But unlike the TT gauge, the “true” radiation gauge apply to general cases, not

just pure wave without source.

Very remarkably, the true radiation gauge also reveals how the TT gauge can be adapted in the presence of source. From Eqs. (12), we can derive

$$h_{0\mu} = -\frac{1}{\bar{\partial}^2} S_{0\mu}, \quad (19a)$$

$$\partial_i h^i_j = -\frac{1}{\bar{\partial}^2} \partial_j T_{00}, \quad (19b)$$

$$h^i_i = -2\frac{1}{\bar{\partial}^2} T_{00}. \quad (19c)$$

Namely, in the presence of source,  $h_{0\mu}$ ,  $h^i_i$ , and  $\partial_i h^i_j$  cannot be all set to zero, but they can indeed be chosen (in our true radiation gauge) to be non-dynamical. Since matter source appear in Eq. (19), we see that the TT “gauge” is rather a combination of field equations and real gauge conditions. These real gauge conditions and (gauge-invariant) field equations can be extracted from Eq. (19) as follows: Act on Eq. (19c) with  $\partial_j$ , and compare the result with Eq. (19b), we get  $\partial_i h^i_j = \frac{1}{2} \partial_j h^i_i$ . Then, act on Eq. (19c) with  $\partial_0$ , act on Eq. (19a) with  $\partial^\mu$ , sum over  $\mu$  from 1 to 3, and use the conservation relation  $\partial^i S_{i0} = \partial_0 T_{00}$ , we get  $\partial_i h^i_0 = \frac{1}{2} \partial_0 h^i_i$ . Thus, (not surprisingly,) we get the full set of the real gauge conditions in Eq. (5) which are solely constrains on  $h_{\mu\nu}$  and independent of the field equations. Then, with these gauge conditions, Eq. (19a) can recover its gauge-invariant form in Eq. (2). Thus we see clearly that the “adapted TT “gauge” in Eq. (19) is indeed a mixture of really independent gauge conditions with original (gauge-invariant) Einstein equations. In retrospect, the original TT gauge in Eq. (4), which is obtained automatically from Eq. (19) by setting source term to zero, is also of a mixed nature. Actually, by repeating the above procedure, we can extract from Eq. (4) the real gauge conditions in Eq. (5), together with gauge-invariant source-free equations for  $h_{0\mu}$ , which are just the  $(0\mu)$ -components of Eq. (2) without source.

It is worthwhile to discuss a “minimum-TT” gauge, which is defined by combining Eqs. (4b) and (4c), and discarding Eq. (4a). In the celebrated canonical formulation of gravity by Arnowitt, Deser, and Misner (ADM) [8], this minimum-TT gauge is regarded as the radiation gauge for gravity, since in this gauge  $h_{ij}$  has only TT components, which in the ADM formulation are the dynamical variables of the gravitational field. We point out that this minimum-TT gauge does not work for general cases either. First, by setting no constraint on coordinate transformation involving only  $x^0$ , it does not fix the gauge completely. Second,

it sets four constraints on purely “spatial” transformations, which admits however only a three-fold gauge freedom. Thus, like the complete TT gauge, the minimum-TT gauge can be imposed only for special cases such as a pure wave, but not general cases. In other words, it is not always possible to transform a general symmetric tensor into purely TT form. Certainly, the combination of Eqs.(19b) and (19c) can be regarded as an adaption of the minimum-TT gauge in the presence of source. But as we just explained, they are rather a combination of true gauge conditions with field equations.

In electrodynamics, the Lorentz gauge  $\vec{\partial} \cdot \vec{A} + \partial_t A^0 = 0$  and radiation gauge  $\vec{\partial} \cdot \vec{A} = 0$  coincide for a static configuration. This property, however, is no longer true in gravitation. This can be seen by comparing Eqs. (1) and (5). They differ by a term  $\partial^0 h^\rho_0 - \frac{1}{2} \partial^\rho h^0_0$ , in which  $\partial^\rho h^0_0$  can be non-zero even for a static case. We can see the crucial difference more directly by looking at the equations of motion. For a static configuration, in the harmonic gauge we have  $\vec{\partial}^2 h_{\mu\nu} = -S_{\mu\nu}$ , while in the radiation gauge we have  $\vec{\partial}^2 h_{0\mu} = -S_{0\mu}$  but

$$\vec{\partial}^2 h_{ij} - \partial_i \partial_j h_{00} = -S_{ij}, \quad (20)$$

which differs significantly from that in the harmonic gauge. In the following, we present explicitly an interesting example for the simplest spherically symmetric solutions. We consider a weak source with a constant mass density  $\rho_0$  and negligible pressure:  $T_{\mu\nu} = \text{diag}\{\rho_0, 0, 0, 0\}$ , and  $S_{\mu\nu} = \frac{1}{2} \text{diag}\{\rho_0, \rho_0, \rho_0, \rho_0\}$ , distributed within a radius  $R$  with total mass  $M$ . In the harmonic gauge, we find the external solution (with  $G$  displayed for clarity)

$$h_{\mu\nu} = \delta_{\mu\nu} \frac{2GM}{r}, \quad (21)$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 + \frac{2GM}{r}\right) d\vec{x}^2, \quad (22)$$

This is just the first-order approximation to the familiar Schwarzschild solution. In the radiation gauge, we find for the external area the same  $h_{0\mu}$  but a distinct  $h_{ij}$ :

$$h_{0\mu} = \delta_{0\mu} \frac{2GM}{r}, \quad h_{ij} = \left(3\delta_{ij} - \frac{x_i x_j}{r^2}\right) \frac{GM}{r}, \quad (23)$$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 + \frac{3GM}{r}\right) d\vec{x}^2 - \frac{GM}{r} \frac{(\vec{x} \cdot d\vec{x})^2}{r^2}. \quad (24)$$

This form appears rather peculiar and has never been discussed before. We will return to its use shortly below.



Besides its own importance as a novel and complete gauge condition for exploring the dynamical structure of gravitation [9], the major relevance of the “true” radiation gauge is for clarifying the long-standing problem of gravitational energy *distribution*. With the increasing opportunity of detecting a gravitational wave and using it as a novel probe into the universe [10], one must now take seriously a long suspended controversial problem, namely, the gauge dependence of energy flow in gravitational radiation. The key obstacle to assigning an unambiguous energy to the gravitational field is the fact that the metric field  $g_{\mu\nu}$  contains spurious gravitational effect associated with coordinate choice. Now that we have found a complete gauge condition which can remove all non-physical degrees of freedom of the gravitational field, the gravitational energy can be safely calculated in this radiation gauge. E.g., it is Eq. (24) rather than Eq. (22) that provides a pertinent metric for calculating the energy density of gravitational field [11]. More importantly, when discussing the distribution of energy flow in gravitational radiation, one should always work in the “true” radiation gauge (5), or equivalently in the TT gauge if one concerns about the radiation zone far away from the source. The latter approach was adopted in Ref. [12]. And we should give (a somewhat academic) reminder that if one concerns about the energy flow near the radiation source, the primitive TT gauge in Eq. (4) cannot be imposed, and only the radiative gauge (5), or equivalently the “adapted TT gauge” in Eq. (19), is appropriate. Another but necessary reminder: Radiative solutions in the harmonic gauge are the easiest to obtain, but must not be directly employed to compute the angular distribution of energy flow [13], since  $g_{\mu\nu}$  in this gauge contain non-physical components which represent a spurious gravitational effect.

*Discussion.*—In this paper we have focused on the weak-field regime and infinitesimal gauge transformations. We now briefly discuss the general cases. We have seen that Eq. (5) appear to be the unique choice for linearized gravity. The extension of Eq. (5) to general cases, however, is not unique. E.g.,  $g^{ij}\Gamma_{ij}^\rho = 0$ ,  $\eta^{ij}\Gamma_{ij}^\rho = 0$ , and  $g^{i\mu}\Gamma_{i\mu}^\rho = 0$  all reduce to Eq. (5) in the linear approximation. It is not easy to tell which choice is better. Beyond the weak-field approximation, the gauge transformation of the gravitational field shows a non-Abelian character:

$$g'_{\mu\nu} = g_{\mu\nu} + (\partial_\lambda g_{\mu\nu})\epsilon^\lambda + g_{\mu\lambda}\partial_\nu\epsilon^\lambda + g_{\nu\lambda}\partial_\mu\epsilon^\lambda. \quad (25)$$

As the lesson learned from non-Abelian gauge theories, for large field amplitude and large

gauge transformation, it should not be expected that the gauge can be completely fixed by simple algebraic and differential constraints in the non-linear theory. The situation for gravity would by no means be simpler, and we leave this highly complicated and non-trivial issue to future studies. For the moment, the only justification we have for preferring  $g^{ij}\Gamma_{ij}^\rho = 0$  is similar to that in defining the harmonic gauge  $g^{\mu\nu}\Gamma_{\mu\nu}^\rho = 0$ . In the harmonic gauge, the coordinate-invariant d'Alembertian  $g^{\mu\nu}D_\mu D_\nu$  (where  $D_\mu$  is the covariant derivative) reduces to the ordinary form:

$$g^{\mu\nu}D_\mu D_\nu\phi = g^{\mu\nu}\partial_\mu\partial_\nu\phi - g^{\mu\nu}\Gamma_{\mu\nu}^\sigma\partial_\sigma\phi = g^{\mu\nu}\partial_\mu\partial_\nu\phi, \quad (26)$$

so that the coordinates  $x^\rho$  are harmonic, in the sense of obeying

$$g^{\mu\nu}D_\mu D_\nu x^\rho = g^{\mu\nu}\partial_\mu\partial_\nu x^\rho = 0. \quad (27)$$

In our radiation gauge  $g^{ij}\Gamma_{ij}^\rho = 0$ , it is the invariant Laplacian  $g^{ij}D_i D_j$  that reduces to the ordinary form:

$$g^{ij}D_i D_j\phi = g^{ij}\partial_i\partial_j\phi - g^{ij}\Gamma_{ij}^\sigma\partial_\sigma\phi = g^{ij}\partial_i\partial_j\phi. \quad (28)$$

Now, the coordinates  $x^\rho$  obey the Laplace equation

$$g^{ij}D_i D_j x^\rho = g^{ij}\partial_i\partial_j x^\rho = 0. \quad (29)$$

Due to the instantaneous feature of the Laplace equation, this can be interpreted as that the coordinate  $x^\rho$  are non-dynamic and do not propagate, thus justifies our choice  $g^{ij}\Gamma_{ij}^\rho = 0$  as the true radiation gauge for the gravitational field. Namely, in this gauge only the gravitational degrees of freedom propagate and the “coordinate wave” is absent.

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- [2] Note that a gauge may not work if its number of constraints does not exceed the actual degrees of freedom, but is doomed to fail if it contains more independent constraints than the actual degrees of freedom.

- [3] In this paper we use frequently such trivial boundary condition at infinity. Note that this does not exclude the discussion of gravitational wave or radiating system, which can be made spatially finite by considering a wavepacket or by letting the radiation occur in a given period. The real case that necessitates a non-trivial boundary condition and requires special care is the universe, which we leave for future studies.
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